

Negative Pressure of Anisotropic Compressible Hall States : Implication to Metrology

Kenzo Ishikawa and Nobuki Maeda

Department of Physics, Hokkaido University, Sapporo 060-0810, Japan
(February 1, 2008)

Electric resistances, pressure, and compressibility of anisotropic compressible states at higher Landau levels are analyzed. The Hall conductance varies continuously with filling factor and longitudinal resistances have huge anisotropy. These values agree with the recent experimental observations of anisotropic compressible states at the half-filled higher Landau levels. The compressibility and pressure become negative. These results imply formation of strips of the compressible gas which results in an extraordinary stability of the integer quantum Hall effect, that is, the Hall resistance is quantized exactly even when the longitudinal resistance does not vanish.

PACS numbers: 73.40.Hm, 73.20.Dx

Recently highly correlated anisotropic states have been observed around half-filled higher Landau levels of high mobility GaAs/AlGaAs hetero-structures.¹⁻⁴ Longitudinal resistance along one direction tends to vanish at low temperature but that of another direction has a large value of order $K\Omega$. The Hall resistance is approximately proportional to the filling factor. Since one longitudinal resistance is finite, the state is compressible. It has been noted also that the current-induced breakdown and collapse of the quantum Hall effect (QHE) at $\nu = 4$ occurs through several steps which implies that compressible gas in quantum Hall system has unusual properties.^{5,6}

Anisotropic stripe states were predicted to be favored in higher Landau levels.^{7,8} Using the von Neumann lattice basis, the present authors found an anisotropic mean field state in the lowest Landau level to have a negative pressure and negative compressibility.^{9,10} Their energies at higher Landau levels were calculated recently by one of the present authors¹¹ and others¹² in the Hartree-Fock approximation. They have lower energies than symmetric states, but their physical properties have not been studied well so that it is not clear if these states agree with the states found by experiments.

In the present work, we study the physical properties of anisotropic mean field states around $\nu = n + 1/2$ and $\nu = n$, where n is an integer. We point out that this mean field state at $\nu = n + 1/2$ has properties described above and can explain the experimental observations of anisotropic states. These states have a negative pressure and negative compressibility and periodic density modulation in one direction. We show that as a consequence of negative pressure, a compressible gas strip is formed in the bulk around $\nu = n$ and a current flows in the strip by a new tunneling mechanism, activation from undercurrent. The current is induced in isolated strip with a temperature dependent magnitude of activation type and it causes a small longitudinal resistance in the system of quantized Hall resistance. This solves a longstanding puzzle of the integer QHE, namely Hall resistance is quantized exactly even if the system has a small finite longitudinal resistance. Collapse phenomena⁵ are shown to be understandable also.

The von Neumann lattice basis^{13,14} is one of the bases for degenerate Landau levels of the two-dimensional continuum space where discrete coherent states of guiding center variables (X, Y) are used and is quite useful in studying QHE because translational invariance in two dimensions is preserved. Spatial properties of extended states and interaction effects were studied in systematic ways. We are able to express exact relations such as current conservation, equal time commutation relations, and Ward-Takahashi identity¹⁵ in equivalent manners as those of local field theory and to connect the Hall conductance with momentum space topological invariant. Exact quantization of the Hall conductance in quantum Hall regime (QHR) in the systems of disorders, interactions, and finite injected current has been proved in this basis. We use this formalism in the present paper as well.

Electrons in the Landau levels are expressed with the creation and annihilation operators $a_l^\dagger(\mathbf{p})$ and $a_l(\mathbf{p})$ of having Landau level index, $l = 0, 1, 2, \dots$, and momentum, \mathbf{p} . The momentum conjugate to von Neumann lattice coordinates is defined in the magnetic Brillouin zone (MBZ), $|p_i| \leq \pi/a$, $a = \sqrt{2\pi\hbar/eB}$. The many body Hamiltonian H is written in the momentum representation as $H = H_0 + H_1$, where

$$H_0 = \sum_{l=0}^{\infty} \int_{MBZ} \frac{d\mathbf{p}}{(2\pi/a)^2} E_l a_l^\dagger(\mathbf{p}) a_l(\mathbf{p}), \quad (1)$$

$$H_1 = \int_{\mathbf{k} \neq 0} d\mathbf{k} \rho(\mathbf{k}) \frac{V(\mathbf{k})}{2} \rho(-\mathbf{k}).$$

Here E_l is the Landau level energy $(\hbar eB/m)(l + 1/2)$ and $V(\mathbf{k}) = 2\pi q^2/k$ for the Coulomb interaction, and the charge neutrality is assumed. In Eq. (1), H_0 is diagonal but the charge density $\rho(\mathbf{k})$ is non-diagonal with respect to l . We call this basis the energy basis. A different basis called the current basis in which charge density becomes diagonal will be used later in computing current correlation functions and electric resistances.

It is worthwhile to clarify the peculiar symmetry of the system described by Eq. (1). The Hamiltonian is invariant under translation in momentum space, $\mathbf{p} \rightarrow \mathbf{p} + \mathbf{K}$,

where \mathbf{K} is a constant vector, which is called the K -symmetry. This symmetry emerges because the kinetic energy is quenched due to the magnetic field. A state which has momentum dependent single particle energy violates the K -symmetry.¹⁶ In the present paper, we study a mean field solution which violates K_y -symmetry but preserves K_x -symmetry. The one-particle energy has p_y dependence in this state.

The compressible gas state is characterized by the following form of expectation values in the coordinate space,

$$U^{(l)}(\mathbf{X} - \mathbf{X}')\delta_{ll'} = \langle a_{l'}^\dagger(\mathbf{X}')a_l(\mathbf{X}) \rangle, \quad (2)$$

where the expectation values are calculated self-consistently in the mean field approximation^{9,11} using H_1 and the mean field $U^{(l)}$. In Figs. 1 and 2, the energy per particle, pressure, and compressibility are presented with respect to the filling factor $\nu = n + \nu'$. As seen in these figures, they become negative. The density is uniform in y -direction but is periodic in x -direction.¹¹ The present anisotropic state could be identified with the stripe structure discussed in Refs. 7 and⁸. We have checked that bubble states discussed in Ref. 7 also have negative pressure and compressibility. These properties may be common in the compressible states of the quantum Hall system.¹⁷

The compressible states thus obtained have negative pressure and are different from ordinary gas. Naively it would be expected that these gas states were unstable. However thanks to the background charge of dopants, a stable state with a negative pressure can exist. Since the pressure is negative, charge carriers compress itself while dopants do no move. Then charge neutrality is broken partly and compressible gas states are stabilized by Coulomb energy. Total energy becomes minimum with a suitable shape which depends on the density of compressible gas.

The bulk compressible gas states are realized around $\nu = n + 1/2$ (called region I), where the Coulomb energy is dominant over negative pressure and a narrow depletion region is formed at the boundary. Its width is determined by the balance between the pressure and the Coulomb force.

The low density compressible gas states are realized around $\nu = n$ (called region II). In this case pressure effect is enhanced relatively compared to Coulomb energy and a strip of compressible gas states is formed as shown in Fig. 3 (a). Real system has disorders by which most electronic states are localized. Let us classify three different regions depending on the relative ratio between localization length ξ at the Fermi energy and the width at potential probe area L_p and the width at Hall probe area L_h . We assume $L_p < L_h$. In the region II-(i), $\xi < L_p$ is satisfied and localized states fill whole system. In the region II-(ii), $L_p < \xi < L_h$ is satisfied and the Hall probe area is filled with localized states but potential probe area is filled partly with compressible gas strip. Finally in the region II-(iii), $L_h < \xi$ is satisfied and whole area are filled partly with compressible gas states. In each case

if localization length is longer than the width of the system, then these localized states are regarded as extended states which behave like compressible gas states with a negative pressure. In the regions II-(ii) and II-(iii), the strip contributes to electric conductance if current flows through the strip. However the strip is unconnected with source drain area. How does the current flow through the strip? This problem has not been studied before. In these regions, extended states below Fermi energy carry non-dissipative current, which we call the undercurrent. See Fig. 3 (b). We show later that the undercurrent actually induces the dragged current.

First we calculate the electric conductance of the bulk compressible states in the region I. It is convenient to use current basis for computing current correlation functions. Field operators a_l and propagator $S_{ll'}$ are transformed from the energy basis to the current basis as,

$$\begin{aligned} \tilde{a}_l(\mathbf{p}) &= \sum_{l'} U_{ll'}(\mathbf{p}) a_{l'}(\mathbf{p}), \\ \tilde{S}_{ll'}(p) &= \sum_{l_1 l_2} U_{ll_1}(\mathbf{p}) S_{l_1 l_2}(p) U_{l_2 l'}^\dagger(\mathbf{p}), \end{aligned} \quad (3)$$

where $U(\mathbf{p}) = e^{-ip_x \xi} e^{-ip_y \eta}$ and (ξ, η) are relative coordinates defined by $(x - X, y - Y)$. In the current basis, the equal time commutation relation between the charge density and the field operators are given by,

$$[\rho(\mathbf{k}), \tilde{a}_l(\mathbf{p})] = -\tilde{a}_l(\mathbf{p}) \delta^{(2)}(\mathbf{p} - \mathbf{k}). \quad (4)$$

Hence vertex part is given by a derivative of inverse of the propagator, $\tilde{\Gamma}_\mu(p, p) = \partial_\mu \tilde{S}^{-1}(p)$, known as Ward-Takahashi identity.¹⁵ The Hall conductance is the slope of the current-current correlation function at the origin and is given by the topologically invariant expression of the propagator in the current basis as^{13,14}

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{24\pi^2} \int \text{tr}(\tilde{S}(p) d\tilde{S}^{-1}(p))^3. \quad (5)$$

This shows that σ_{xy} is quantized exactly in QHR where the Fermi energy is located in the localized state region. Now Fermi energy is in the compressible state band region and σ_{xy} is not quantized. For the anisotropic states, the inverse propagator is given by $S^{-1}(p)_{ll'} = \{p_0 - (E_l + \epsilon_l(p_y))\} \delta_{ll'}$ where $\epsilon_l(p_y)$ is the one-particle energy.^{9,11} $S(p)$ has no topological singularity and its winding number vanishes. Hence the topological property of the propagator in the current basis, $\tilde{S}(p)$, is determined solely by the unitary operator $U(\mathbf{p})$, and the Hall conductance is written as,

$$\begin{aligned} \sigma_{xy} &= \frac{e^2}{h} \frac{1}{4\pi^2} \int dp \epsilon^{ij} \text{tr} [S(p) U^\dagger(\mathbf{p}) \partial_i U(\mathbf{p}) U^\dagger(\mathbf{p}) \partial_j U(\mathbf{p})] \\ &= \frac{e^2}{h} (n + \nu'). \end{aligned} \quad (6)$$

To obtain the final result in the above equation, we assumed that the Landau levels are filled completely up to

n th level and $(n+1)$ th level is filled partially with filling factor ν' . The Hall conductance is proportional to the total filling factor.

The longitudinal conductance in x-direction, σ_{xx} , vanishes since there is no empty state in this direction. If a momentum is added in x-direction, one particle should be lifted to a higher Landau level. There needs a finite energy and σ_{xx} vanishes. The longitudinal conductance in y-direction, σ_{yy} , does not vanish. One particle energy has a dependence on only p_y , and the system is regarded as one dimensional. One dimensional conductance is given by Buttiker-Landauer formula¹⁸. We have thus,

$$\sigma_{yy} = e^2/h, \sigma_{xx} = 0. \quad (7)$$

The Hall conductance, Eq. (6), and the longitudinal conductances, Eq. (7), agree with the experimental observations of anisotropic states around $\nu = n + 1/2$ ($n \geq 1$).

Next we study the low density region, region II. In the first region, II-(i), whole area is filled with localized states, hence from the formula Eq. (5), the Hall conductance is quantized exactly. The longitudinal resistances vanish. We have

$$\sigma_{xy} = (e^2/h)n, \sigma_{xx} = \sigma_{yy} = 0. \quad (8)$$

This corresponds to standard QHR.

In the regions II-(ii) and II-(iii), a compressible strip bridges one edge to the other edge. Tunneling combined with an interaction causes the dragged current in the strip. The conductance due to the tunneling mechanism can be calculated by a current-current correlation function shown in Fig. 4. The two-loop diagram is the lowest order contribution. The dragged current flows in the compressible strip at potential probe area in the region II-(ii), and at Hall probe area in the region II-(iii).

In the region II-(ii), the Hall probe area is filled with only localized states. Hence the Hall conductance is quantized exactly. The potential probe area has a finite longitudinal resistance due to an electric current in the strip. The electric current which flows in the strip makes the strip area to have a finite temperature. We have thus the exactly quantized Hall resistance and a small longitudinal resistance in this region as

$$R_{xy}^{-1} = (e^2/h)n, R_{xx}^{-1} = (e^2/h)\varepsilon. \quad (9)$$

The small parameter ε is proportional to the activation form $\exp[-\beta(\Delta + mv^2/2)]$, where Δ is the energy gap between the Fermi energy and the lower Landau level, β is the inverse temperature at strip area, and v is the average velocity of the undercurrent states. The additional term $mv^2/2$ to Δ comes from the Galilean boost. This region has not been taken into account in the metrology of QHE.¹⁹

In the region II-(iii), whole area is filled with compressible states. Hence from Eq. (6), the Hall conductance is given by unquantized value and the longitudinal resistance becomes finite. In this case we have

$$R_{xy}^{-1} = (e^2/h)(n + \varepsilon'), R_{xx}^{-1} = (e^2/h)\varepsilon. \quad (10)$$

That is, QHE is collapsed.⁵ ε' has the same temperature dependence as ε .

The localization length and mobility edge depend on injected current in the real system. In a small current system, localization lengths are small in a mobility gap and corresponds to QHR (region II-(i)). In an intermediate current system, they become larger and the strip is formed in potential probe area first (region II-(ii)) and in Hall probe area second (region II-(iii)). QHE is collapsed in this region. In a larger current system, localization lengths become even larger, and whole system is filled with extended states. QHE is broken down in this region. This is consistent with Kawaji et al.'s recent experiments and proposal.⁵

In summary, we have shown that the anisotropic mean field states have unquantized Hall conductance, huge anisotropic longitudinal resistance, negative pressure, and negative compressibility. These electric properties are consistent with the recent experiments of the anisotropic compressible Hall state and of collapse phenomena. Negative pressure of these states does not lead to instability but instead leads to a formation of a narrow strip of compressible gas states if its density is low and formation of the bulk compressible gas with the depletion region if its density is around the half-filling. Consequently in the system of low density compressible gas states, the Hall resistance is kept in the exactly quantized value even though the longitudinal resistance is finite. This longstanding puzzle was solved from the unusual property of compressible Hall gas, namely negative pressure. Hence it plays important roles in the metrology of the QHE.

Authors would like to thank Y. Hosotani and T. Ochiai for useful discussions. One of the present authors (K. I.) also thanks S. Kawaji and B. I. Shklovskii for fruitful discussions. This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sports, and Culture, the Grant-in-Aid for Scientific Research on Priority area (Physics of CP violation) (Grant No. 12014201), and the Grant-in-aid for International Science Research (Joint Research 10044043) from the Ministry of Education, Science, Sports and Culture, Japan.

¹ M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82**, 394 (1999).

² R. R. Du, D. C. Tsui, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Solid State Commun. **109**, 389 (1999).

³ W. Pan, R. R. Du, H. L. Stormer, D. C. Tsui, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **83**, 820 (1999).

⁴ M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **83**, 824 (1999).

⁵ S. Kawaji, J. Suzuki, T. Shimada, H. Iizuka, T. Kuga, and T. Okamoto, J. Phys. Soc. Jpn. **67**, 1110 (1998); S. Kawaji, H. Iizuka, T. Kuga, and T. Okamoto, Physica **B256-258**, 56 (1998).

⁶ J. Weis, Y. Y. Wei, and K. v. Klizing, Physica **B256-258**, 1 (1998); I. I. Kaya, G. Nachtwei, K. v. Klizing, and K. Eberl, *ibid.* **256-258**, 8 (1998).

⁷ A. A. Koulakov, M. M. Fogler, and B. I. Shklovskii, Phys. Rev. Lett. **76**, 499 (1996); M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. B **54**, 1853 (1999).

⁸ R. Moessner and J. T. Chalker, Phys. Rev. B **54**, 5006 (1996).

⁹ K. Ishikawa, N. Maeda, and T. Ochiai, Phys. Rev. Lett. **82**, 4292 (1999).

¹⁰ Actually, the negative compressibility was observed by J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **68**, 674 (1992).

¹¹ N. Maeda, Phys. Rev. B **61**, 4766 (2000).

¹² T. Jungwirth, A. H. MacDonald, L. Smrcka, and S. M. Girvin, Phys. Rev. B **60**, 15574 (1999); T. Stanescu, I. Martin, and P. Phillips, Phys. Rev. Lett. **84**, 1288 (2000);

¹³ N. Imai, K. Ishikawa, T. Matsuyama, and I. Tanaka, Phys. Rev. B **42**, 10610 (1990); K. Ishikawa, Prog. Theor. Phys. Suppl. **107**, 167 (1992); K. Ishikawa, N. Maeda, K. Tadaki, Phys. Rev. B **54**, 17819 (1996).

¹⁴ K. Ishikawa, N. Maeda, T. Ochiai, and H. Suzuki, Physica (Amsterdam) **4E**, 37 (1999); Phys. Rev. B **58**, 1088 (1998); *ibid.* **58**, R13391 (1998).

¹⁵ J. C. Ward, Phys. Rev. **78**, 1824 (1950); Y. Takahashi, Nuovo Cimento **6**, 370 (1957).

¹⁶ The K-symmetry was originally discussed in the context of the composite Fermion theory. In the composite Fermion theory, the concept of compressibility is controversial, see V. Pasquier and F. D. M. Haldane, Nucl. Phys. **B516**, 719 (1998); B. I. Halperin and A. Stern, Phys. Rev. Lett. **80**, 5457 (1998); G. Murthy and R. Shankar, *ibid.* **80**, 5458 (1998).

¹⁷ A. L. Efros, Solid State Commun. **65**, 1281 (1988).

¹⁸ M. Buttiker, Phys. Rev. B **38**, 9375 (1988); R. Landauer, IBM J. Res. Dev. **1**, 223 (1957).

¹⁹ K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980); S. Kawaji and J. Wakabayashi, in *Physics in High Magnetic Fields*, edited by S. Chikazumi and N. Miura (Springer-Verlag, Berlin, 1981); F. Delahaye, Metrologia, **26**, 63 (1989).

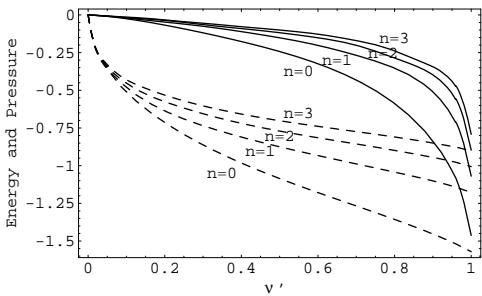


Fig 1. Energy per particle (dashed lines) in the unit of q^2/a and pressure (solid lines) in the unit of q^2/a^3 for $\nu = n + \nu'$, $n = 0, 1, 2$, and 3 .

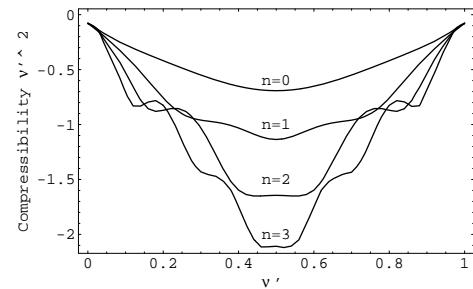


Fig 2. Compressibility times ν'^2 in the unit of a^3/q^2 for $\nu = n + \nu'$, $n = 0, 1, 2$, and 3 .

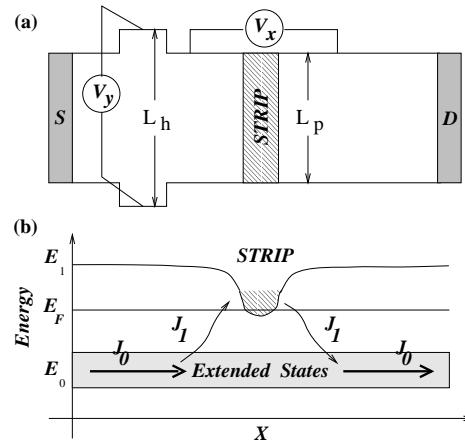


Fig 3. (a) Schematic view of a Hall bar in the region II-(ii). Electric current is injected from S to D. (b) Sketch of the extended states carrying the undercurrent J_0 and the compressible state carrying the dragged current J_1 at the strip area in the energy and the x-position.

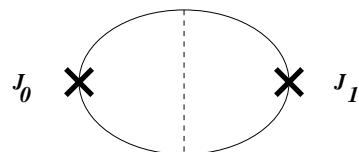


Fig 4. Feynman diagram for the current-current correlation function. J_0 is the undercurrent carried by the extended states and J_1 is the dragged current carried by the compressible states in the strip area. The dashed line stands for an interaction effect.